Bistable and tristable soliton switching in collinear arrays of linearly coupled waveguides

Alejandro B. Aceves* and Marco Santagiustina†
Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912
(Received 19 February 1997)

The soliton switching in an array of three, linearly coupled waveguides is investigated. Supported by a variational model, we show that a novel nonlinear switching among stable localized states is possible in long couplers. The numerical integrations of the governing equations confirm this interesting phenomenon. These results can be extended to asymmetrical nonlinear directional couplers, where the asymmetry is in the nonlinear coefficients. The tristable operational mode is investigated too and the model furnishes valuable indications in order to realize it. The model also allows a physical insight into the more general problem of steering in arrays with a large number of waveguides. Finally, an exact, antisymmetric solution of the governing equations of the three-waveguide array is also investigated, because it shows dynamical properties that might be useful in all-optical switching. [S1063-651X(97)08007-0]

PACS number(s): 42.65.Tg, 42.65.Wi

INTRODUCTION

Soliton switching in nonlinear directional couplers (NLDC's) [1] is regarded as a possible mechanism for all-optical signal processing [2] and hence it has been the object of experimental [2] as well as analytical and numerical investigations [3–8].

The switching of cw signal in arrays of more than two waveguides [9–11], such as multicore fibers [12] or coupled semiconductor waveguides [13], is also under current investigation. This interest stems from the fact that sharper switching characteristics are expected in these devices [10]. Unfortunately, the growth of the energy threshold for the switching, as the number N of waveguides increases, and the lack of analytical tools to predict the dynamics have hampered their effective exploitation so far. Eventually, especially in long couplers, chaotic dynamics has been reported to arise as soon as $N \geqslant 3$ [14,15].

Hence the nonlinear switching of cw signals as well as of solitonlike pulses has been numerically investigated for limited sets of initial conditions in the case of collinear half-beat length three-waveguide arrays [14,15]. Those results show the predicted sharper switching characteristic, when a pulse is launched in one of the side waveguides and no pulses are present in the remaining waveguides.

Though this is the simplest configuration, one might expect a richer dynamics to appear if other initial conditions are used and this paper is aimed to be a step in the direction of better understanding the behavior of these systems and their potential use in all-optical signal processing.

Besides the coupling dynamical regimes, waveguide arrays possess a large family of steady-state solutions [16,17]. Particularly interesting are the so called localized states, i.e., those solutions for which most of the energy is concentrated in a single waveguide of the array [18–20]. Many of these solutions have a stable branch and, as we show elsewhere in detail [21], their appearance can be explained as a crossing of a separatrix in the phase space of a dynamical system in the pulse parameters derived from a reduced Lagrangian approximation. In fact, if the energy is increased, orbits move away from the region of coupling towards another region of almost constant pulse amplitude and rotating phase, i.e., localized states.

This transition is similar to that which occurs in the NLDC at the switching threshold [1,4,6], though an important distinction must be noted. In fact, for the NLDC there exist two symmetric localized states (the energy being concentrated in one of the waveguides) each characterized by the same energy threshold because of the intrinsic symmetry of the device. However, if the number of waveguides is greater than 2, several new localized states appear, each characterized by an energy threshold which, in principle, may be different from that of other solutions. The potential of multistable operation of these devices is thus clear, if some perturbation is found that, when applied, the system moves from an initial high energy mode to a final stable state at lower energy. The net loss in such a scenario would account for radiation, which has been already indicated as a major feature in the context of the transition from unstable to stable solutions in the NLDC [22] and of the relaxation oscillations while reaching final state both in NLDC's [22,23] and three-waveguide arrays [21,24].

In this paper we will concentrate on the case of a three-waveguide collinear array. First, we will show that by exploiting the energy gap among two different stable localized solutions, it is possible to obtain a bistable switching from this device. Note that this behavior is very different, from the physical viewpoint, from that of the NLDC, where the stable state can be broken only to initiate the typical linearlike coupling dynamics. This stems from the fact that in the NLDC there is no energy gap, as said, among the states. This condition is necessary, as we will show, in order to compensate the loss due to radiation, during the state commutation.

Eventually, if symmetrical initial conditions are applied, it is worth noting that the three-waveguide device is equivalent to an asymmetrical NLDC [25] and bistability would occur

*Permanent address: Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131.

†Present address: Departament de Fisica, Universitat de les Illes Balears, Palma de Mallorca, Spain.

too. In particular, our case would correspond to a NLDC where the cores have equal linear properties and a mismatch in the nonlinearity. Recently the chance of bistability in couplers which present asymmetries in the linear properties of the cores has been suggested too [26]. Moreover, we will explore possible tristable soliton switching, which is plausible in such a device. Analytical considerations will provide us useful indications in order to achieve this goal. Our approach will also give enough evidence to explain the reason why the steering of localized solutions has proved to be hard to achieve in arrays with a large number of waveguides unless strong actions are taken [11,18–20].

We will also study the dynamical evolution of an exact, antisymmetric stationary solution of the governing equation. In fact, the results of the analytical approach give strong indications that a slow (in the propagation distance) dynamics is associated to this input condition. The numerical solutions final stable state is observed if energy is above the predicted threshold.

The paper is organized as follows: first we will present the governing equations and their reduction, by means of a reduced variational approach, to a simpler set of coupled ordinary differential equations (ODE's), where the variables are the amplitude and the phase of the pulses.

In the case of symmetrical initial conditions this model provides excellent estimates of the energy thresholds for two possible localized solutions. The bistable switching of these two solutions, predicted by the analysis, is then confirmed by means of the numerical solutions of the partial differential equations (PDE's) which govern the full dynamics.

Supported by the numerical solutions, physical arguments, and the analytical findings we will then search for tristable operation. As an additional result, we will also gain valuable information about the general problem of steering in large arrays of coupled waveguides.

GOVERNING EQUATIONS

The optical pulse propagation in a system of three identical, collinear, linearly coupled waveguides with a Kerr-type nonlinear index of refraction, can be described by the following set of equations [15]:

$$iq_z^{(1)}+\tfrac{1}{2}q_{tt}^{(1)}+|q^{(1)}|^2q^{(1)}-K(q^{(2)}-2q^{(1)})0,$$

$$iq_z^{(2)}+\tfrac{1}{2}q_{tt}^{(2)}+|q^{(2)}|^2q^{(2)}-K(q^{(1)}+q^{(3)}-2q^{(2)})=0,$$

$$iq_z^{(3)}+\tfrac{1}{2}q_{tt}^{(3)}+|q^{(3)}|^2q^{(3)}-K(q^{(2)}-2q^{(3)})=0. \tag{1}$$

The indices z and t indicate differentiation with respect to the propagation distance z, scaled to the dispersion length $L_D=t_0^2/|k''|$ (k'' is the group velocity dispersion), and the time t, scaled by t_0 ($T_{\mathrm{FWHM}}=1.763t_0$). The parameter K is equal to CL_D, where C is the coupling coefficient (m^{-1}) and finally the fields $q^{(n)}$ are related to the physical intensities $|\psi^{(n)}|^2$ by $q^{(n)}=(2\pi n_2 L_D/\lambda_0)^{1/2}\psi^{(n)}$. Equations (1) can also be considered as the Euler-Lagrange equations

$$\frac{\partial}{\partial z}\frac{\partial L}{\partial q_z^{(n)*}}+\frac{\partial}{\partial t}\frac{\partial L}{\partial q_t^{(n)*}}-\frac{\partial L}{\partial q^{(n)*}}=0 \tag{2}$$

of the Lagrangian density

$$L=\sum_{n=1}^{3}L_n+L^{\mathrm{int}}, \tag{3}$$

where

$$L_n=\frac{i}{2}(q^{(n)*}q_z^{(n)}-q^{(n)}q_z^{(n)*})-\frac{1}{2}|q_t^{(n)}|^2+\frac{1}{2}|q^{(n)}|^4 \tag{4}$$

and

$$L^{\mathrm{int}}=-K\left(q^{(1)*}q^{(2)}+\text{c.c.}+q^{(3)*}q^{(2)}+\text{c.c.}-\sum_{n=1}^{3}|q^{(n)}|^2\right), \tag{5}$$

where c.c. means the complex conjugate of the previous term in the formula.

ANALYSIS OF THE BISTABLE SOLITON SWITCHING

In this section we start by introducing a simplified model of the nonlinear dynamics of the governing equations, by means of a variational approach. In fact, based on the Lagrangian formulation, it is possible to derive a dynamical system for the relevant pulse parameters by means of a reduced variational problem. As has been shown in several papers [4–6,8], this is a very powerful method that gives useful information on the behavior of the system, even though it is a severe reduction from an infinite dimensional dynamical system to a finite and typically low dimensional one, especially when a small number of parameters are used [8]. In order to verify the predictions obtained from the variational approach, one needs to compare the results with the dynamics of the full system (1). We do so by numerically integrating the system of Eqs. (1) by means of a split-step, finite-difference algorithm. Transparent boundary conditions [27] have been imposed in order to allow radiation, which may travel at a speed different from the soliton part, to flow outside the computational domain. We emphasize though that the merits of the Lagrangian approach are that it gives analytical insight into the behavior of the system and defines the states as well as determines the parameter values for the switching to occur.

We begin by calculating a reduced Lagrangian density $\mathcal{L}=\int_{-\infty}^{+\infty}L\,dt$, by integrating over a set of trial functions of the kind $f_n(z,t)=A_n\mathrm{sech}(A_nt)\exp(ip_n)$, $n=(1,2,3)$ [7], thus obtaining a set of ODE's for the parameters $A_{1,2,3}(z)$ and $p_{1,2,3}(z)$.

These equations are invariant with regard to phase shifts and thus we may consider only the differential phases $p_-=p_2-p_3$, $p_+=p_2-p_1$ and the system reduces to five coupled equations, which read

$$\frac{dA_1}{dz}=KA_1A_2\sin(p_+)I(A_1,A_2),$$

$$\frac{dA_2}{dz}=-KA_2[A_1\sin(p_+)I(A_1,A_2)+A_3\sin(p_-)I(A_2,A_3)],$$

$$\frac{dA_3}{dz}=KA_2A_3\sin(p_-)I(A_2,A_3),$$

$$\frac{dp_+}{dz}=\frac{1}{2}(A_2^2-A_1^2)+K(A_2-A_1)\cos(p_+)I(A_1,A_2)$$
$$+A_1A_2\cos(p_+)\left[\frac{\partial I(A_1,A_2)}{\partial A_1}-\frac{\partial I(A_1,A_2)}{\partial A_2}\right]$$
$$-KA_3\cos(p_-)I(A_2,A_3)$$
$$-KA_2A_3\cos(p_-)\frac{\partial I(A_2,A_3)}{\partial A_2},$$

$$\frac{dp_-}{dz}=\frac{1}{2}(A_2^2-A_3^2)+K(A_2-A_3)\cos(p_-)I(A_2,A_3)$$
$$+KA_2A_3\cos(p_-)\left[\frac{\partial I(A_2,A_3)}{\partial A_3}-\frac{\partial I(A_2,A_3)}{\partial A_2}\right]$$
$$-KA_1\cos(p_+)I(A_1,A_2)$$
$$-KA_1A_2\cos(p_+)\frac{\partial I(A_1,A_2)}{\partial A_2}, \tag{6}$$

where

$$I(A_n,A_m)=\int_{-\infty}^{+\infty}\mathrm{sech}(A_nt)\mathrm{sech}(A_mt)dt. \tag{7}$$

If we impose the symmetry condition $q^{(1)}(z,t)=q^{(3)}(z,t)$ and the fact that the governing system has an additional conserved quantity, namely, the mass (physically the energy)

$$E=\sum_{n=1}^{3}\int_{-\infty}^{\infty}|q^{(n)}(t,z)|^2dt=2\sum_{n=1}^{3}A_n, \tag{8}$$

eventually the problem can be reduced to a Hamiltonian system of two equations for the energy fraction of the central waveguide $a=2A_2/E$ and the phase difference $p=p_+=p_-$ (see also [21,24]).

$$\frac{da}{dZ}=\frac{\partial h}{\partial p},$$

$$\frac{dp}{dZ}=-\frac{\partial h}{\partial a}, \tag{9}$$

where the effective Hamiltonian is

$$h(a,p)=\beta(1-8a+8a^2+8a^3)+4a(a-1)\cos(p)I(a), \tag{10}$$

where, from Eqs. (7) and (8), $I(a)$ is now

$$I(a)=\int_{-\infty}^{+\infty}\mathrm{sech}\left[\frac{aE}{2}t\right]\mathrm{sech}\left[\frac{(1-a)E}{4}t\right]dt \tag{11}$$

and finally $\beta=E/(32K)$, $Z=KEz/4$.

Note that when the symmetry is used, the model is also equivalent to that of a coupler with unequal nonlinear coefficients [25]. We want to stress this fact because it discloses opportunities of novel devices, very interesting from the viewpoint of all-optical signal processing, as we will show below.

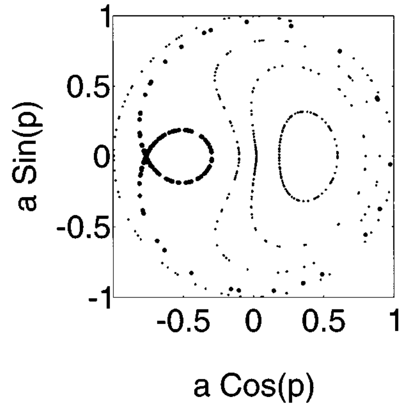

FIG. 1. Phase portrait of the orbits in the phase plane $(a\cos(p), a\sin(p))$ for $K=1$, $E=\sqrt{40}$ (from the variational approach). The large dots represent a separatrix, and their intersection a saddle point. Note the region close to the boundary, which implies the existence of localized states.

As said, we are interested in localized solutions of Eqs. (1), i.e., pulse propagation with large, stable amplitude and a phase linearly varying in z.

Among the symmetric solutions of the reduced dynamical system (6) [or equivalently Eq. (9)] we find $A_1=A_3=0$, $A_2=E/2$, $p_+=p_-=A_2^2/2z$ (i.e., $a=1$, $p=p_+$), which we define as the state ''1,'' and $A_1=A_3=E/4$, $A_2=0$, $p_+=p_-=-A_1^2/2z$ (i.e., $a=0$, $p=p_+$) as state ''0.'' The first is stable as soon as $E>E_{cr1}=\sqrt{8\pi K}$; in fact for values above this threshold there exists a region of orbits for which the coupling dynamics does not occur, as we can observe in the phase portrait of the system (9), which is presented in Fig. 1 [21]. Here, the region between the separatrix and the circle of radius 1 describes orbits of localization. Physically this is the regime where the nonlinearity overwhelms the coupling and thus the energy is trapped into the central waveguide. This can be seen directly from Eqs. (6); upon substitution of the solution we find that the first term in the phase equations (p_+,p_-), which accounts for the nonlinear part of Eqs. (1), equals, in absolute value, the terms depending on the linear coupling (K) exactly at threshold.

Similarly, for the state ''0,'' we find that the energy threshold for stability is $E_{cr0}=\sqrt{64\pi K}$ [21]. Note that this energy is $2\sqrt{2}$ times larger than the previous one, i.e., it requires each pulse of the side waveguides to be $\sqrt{2}$ times more energetic.

These two solutions can be mapped into equivalent solutions of the original problem. The state ''1'' corresponds to a solution of the family having to first approximation the following form:

$$q^{(2)}(z,t)=\eta\mathrm{sech}(\eta t)\exp\left(i\frac{\eta^2}{2}z\right), \tag{12}$$

$$q^{(1,3)}(z,t)=\frac{1}{2\eta}[e^{\eta t}\ln(1+e^{-2\eta t})+e^{-\eta t}\times\ln(1+e^{2\eta t})]\exp\left(i\frac{\eta^2}{2}z\right)$$

for which η is the family parameter. This is, in fact, an approximate solution of Eqs. (1) under the assumption that most of the energy is localized into the central waveguide and only linear waves are propagating in the sides (i.e., $\eta>1$ [19,20]). The state ''0'' is to first approximation a solution of two uncoupled nonlinear Schrödinger (NLS) equations and a linear equation whose forcing term is the sum of the solutions of the other equations and thus we find

$$q^{(1)}(z,t)=q^{(3)}(z,t)=\eta\mathrm{sech}(\eta t)\exp\left(i\frac{\eta^2}{2}z\right), \quad (13)$$

$$q^{(2)}(z,t)=\frac{1}{\eta}[e^{\eta t}\ln(1+e^{-2\eta t})+e^{-\eta t}\ln(1+e^{2\eta t})]\times\exp\left(\frac{i\eta^2}{2}z\right),$$

where again η is a variable parameter.

In comparing Eqs. (12) and (13) with the trial functions, we observe that these functions fail to capture the common width $1/\eta$ of all three pulses in the localized states. On the other hand, as one can see throughout the remainder of the paper, these only introduce small quantitative errors, while the overall qualitative features of the global dynamics are well captured by the variational approach. The quantitative error is in general small, because the failure in capturing the correct width in the regime of localization is only on those pulses where the amplitude is almost negligible. If one derives a variational formulation based on trial functions of the form $f_n(z,t)=A_n\mathrm{sech}(Bt)\exp(ip_n)$, one obtains stationary solutions of the reduced variational equations corresponding to Eqs. (12) and (13); on the other hand, the additional parameter B makes the general dynamical system more complex, losing the ability to make predictions on the general dynamics.

The existence of the energy gap among stability thresholds of solutions (12) and (13), as indicated by the variational approach, is the key point of the bistability. Let us set the initial condition in the state ''0,'' which has the larger threshold for stability, with an energy slightly above E_{cr0}. In this case the coupling dynamics is reduced to small oscillations around a stable solitonlike solution, as confirmed by the numerical solution presented in Fig. 2. Note that almost no radiation is emitted in this case. However, if the energy decreases just below threshold, the state becomes unstable and the energy may transfer to the central fiber. Though some part of it is lost in the process, since radiation is produced, nonetheless, given that $E_{cr0}>E_{cr1}$, one may expect that the stable state ''1'' would form as the result of the coalescence of the incoming fields. In fact, in Fig. 3 we show this phenomenon, thus demonstrating the bistable, energy-controlled soliton switching. Note that the predicted threshold is just 2% different from the numerically found value, which is quite a remarkable result. The energy driven into the central

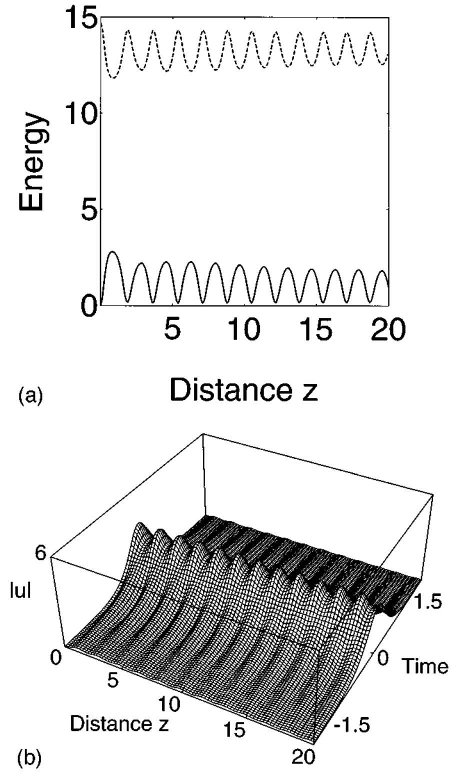

FIG. 2. (a) Pulse energy as a function of propagation distance. The solid (dashed) line represents the pulse energy in the central waveguide (the total energy in the side waveguides). The initial conditions are $E=8\sqrt{\pi K}+0.5$, $K=1$, $q^{(n)}=A_n\mathrm{sech}(A_n,t)$, $A_1=A_3=0.999E/4$, $A_2=0.001E/2$. (b) Pulse evolution of $u=q^{(1)}=q^{(3)}$ for the same solution.

waveguide is very high and the final state is formed only after many relaxation oscillations, as found in the case of the NLDC [22,23], to which both the coupling and the pulse-radiation interaction contribute [21]. In order to verify that the system is really oscillating around a solitonlike solution we performed the same numerical integration though setting $K=0$ for $z>10$, thus decoupling the waveguides: the result is presented in Fig. 4.

This device shares this noteworthy bistable property with the asymmetrical NLDC's, for which the effect should be attained at lower input energy. In fact, the case we presented is equivalent to an asymmetric NLDC with a ratio of 2 among the core nonlinearities. This condition can be obtained in fiber couplers by weakly doping one of the cores with semiconductors; this induces a large enhancement on the nonlinearity [28], due to the large difference in the material nonlinear susceptibilities [29], but a weak change in the linear properties, according to the Maxwell-Garnet theory (see [30] for a review on semiconductor doped glasses). At the limit, when no nonlinear mismatch exists, the asymmetry disappears and the NLDC states have equal stability threshold. Thus by means of a weak doping it should be possible to tune the energy gap $E_{cr0}-E_{cr1}$ to the minimal value for bistable switching to occur in spite of radiation losses. Numerical results indicate that the loss is small compared to the smallest of the two thresholds and thus the total needed energy is not much larger than the threshold energy for switching in a symmetrical NLDC

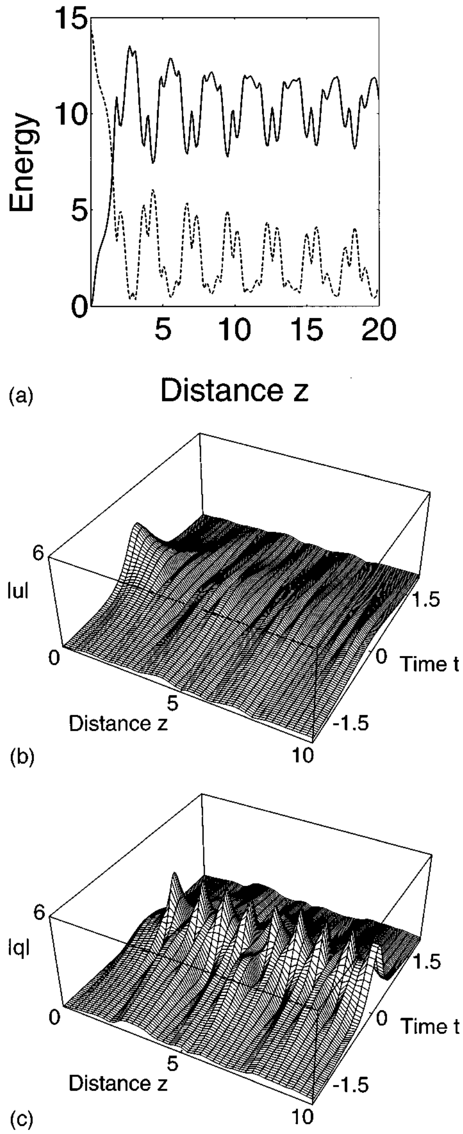

FIG. 3. (a) The same as in Fig. 2(a) but with initial energy $E=8\sqrt{\pi K}+0.25$. The energy is below threshold and energy transfer occurs. (b) The same as for Fig. 2(b). Only weak radiation still couples back, after the energy transfer has taken place. (c) Stable localization of energy into the central waveguide $q=q^{(2)}$. Much radiation is emitted due to the large amount of energy transferred; this causes the large oscillations of the new stable state.

which, in our notation, is $\sqrt{24K}$ [4,6]. Notice that this value is very close to $E_{cr1}=\sqrt{8\pi K}$; that is, the lowest energy level, in spite of the asymmetry, is close to that of the NLDC. This result is confirmed in previous work [15], where a comparison among the switching characteristics was done.

We might also calculate, as an example, the physical energy threshold corresponding to the normalized value $E_{cr1}=\sqrt{8\pi K}$. Its value is given by

$$E_t=A_{\mathrm{eff}}\sum_{n=1}^{3}\int_{-\infty}^{\infty}|\psi^{(n)}|^2 d\tau, \tag{14}$$

where A_{eff} is the effective area of the waveguides, $|\psi^{(n)}|^2$ are the intensities, and τ is the time. By recalling the relations among dimensionless and physical variables we have

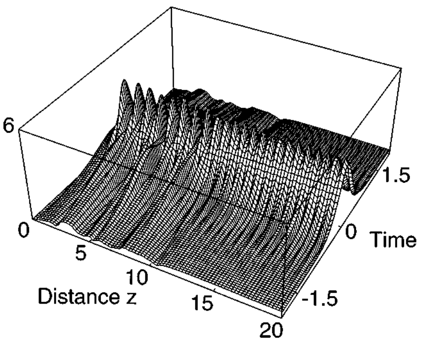

FIG. 4. The same as in Fig. 3(c), but setting $K=0$ beyond $z=10$. By decoupling the waveguides we can speed up the convergence towards the final solution.

$$E_t=\frac{A_{\mathrm{eff}}\lambda_0 t_0}{2\pi n_2 L_D}\sum_{n=1}^{3}\int_{-\infty}^{\infty}|q^{(n)}|^2 dt. \tag{15}$$

At threshold, the sum of the integrals of Eq. (15) is given by Eq. (8) and is equal to $E_{cr1}=\sqrt{8\pi K}$. Thus for $K=1$ we finally find

$$E_t=\left(\frac{2}{\pi}\right)^{1/2}\frac{A_{\mathrm{eff}}\lambda_0|k''|}{n_2 t_0}. \tag{16}$$

If the device is a multicore fiber we can use the following values: $k''=-20$ ps^2/km, $n_2=3.25\times10^{-20}$ m^2/W, $\lambda_0=1.55$ μm, and an effective core area $A_{\mathrm{eff}}\sim 64$ μm^2; therefore when $t_0=1$ ps we get an estimated value of $E_t=1.5$ pJ. The full width at half maximum of the pulse intensity (T_{FWHM}) is this example is given by $T_{\mathrm{FWHM}}=1.763t_0/(E_{cr1}/2)\sim 0.7$ ps; in fact $E_{cr1}/2$ is the normalized soliton width and 1.763 a shape coefficient for the squared hyperbolic secant. The coupling coefficient is $C=1/L_D=0.32$ m^{-1}, which is compatible with reported experimental values [12].

ANALYSIS OF THE TRISTABLE SWITCHING AND STEERING IN LARGE ARRAYS

So far we have found that bistable soliton switching is possible for arrays of three coupled waveguides or NLDC with unequal nonlinearities. However, the former device possesses a larger variety of stable localized states and tristable operation could be a feature too.

It is obvious that this operational mode cannot be accomplished solely by means of symmetrical solutions, a condition which actually reduces the number of possible steady states. The simplest choice would be to use three states in which almost the whole energy is concentrated in one waveguide. We already know the threshold of stability of solution with the energy localized in the central waveguide (at least in the initially symmetrical regime). The other two are new stable localized solution whose existence is easily demonstrated. In fact, if the energy is concentrated on one side, the opposite waveguide does not affect its linear stability, as we note by inspection of Eqs. (1). Thus the array can be actually reduced to the NLDC; this is confirmed by the results of Ref. [15] where this type of asymmetrical initial conditions were used. The two solutions are of the kind [19]

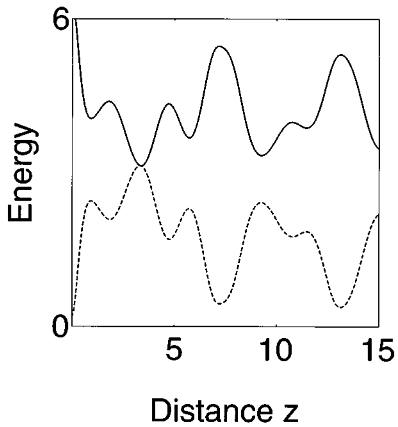

FIG. 5. Energy evolution for an initial condition with a broader distribution. The data of this numerical solution are $E=\sqrt{45}$, $K=1, q^{(n)}=A_n\mathrm{sech}(A_n t)$, $A_1=A_3=0.06E/4$, $A_2=0.94E/2$. In agreement with the model, a localized state is formed.

$$q^{(n)}(z,t)\approx \eta\mathrm{sech}(\eta t)\exp\left(i\frac{\eta^2}{2}z\right),$$

$$q^{(2)}(z,t)\approx \frac{1}{2\eta}[e^{\eta t}\ln(1+e^{-2\eta t})+e^{-\eta t}\ln(1+e^{2\eta t})]$$

$$\times\exp\left(i\frac{\eta^2}{2}z\right),$$

$$q^{(m)}(z,t)=O\left(\frac{1}{\eta^3}\right)f(\eta t)\exp\left(i\frac{\eta^2}{2}z\right) \tag{17}$$

for the (n,m) pairs (1,3) and (3,1). Thus the energy threshold for stability of these modes is expected to be very close to that of the NLDC; our and previous (see Ref. [15]) numerical solutions have confirmed this fact.

In principle the tristable switching would be possible, since a small energy gap exists among the stability thresholds of these states, and might be accomplished by a symmetry breaking perturbation similar to that applied for steering in large arrays [11,18–20]. This difference amounts to

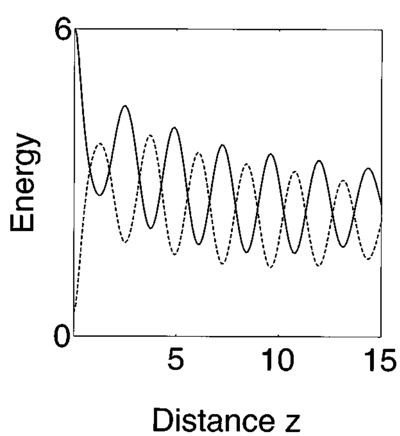

FIG. 6. The same as Fig. 5 but with a slightly different distribution of the same energy, i.e., $A_1=A_3=0.1E/4$, $A_2=0.9E/2$. The initial condition is not stable and localization is lost; the onset of a coupling dynamics is observed, as predicted by the model.

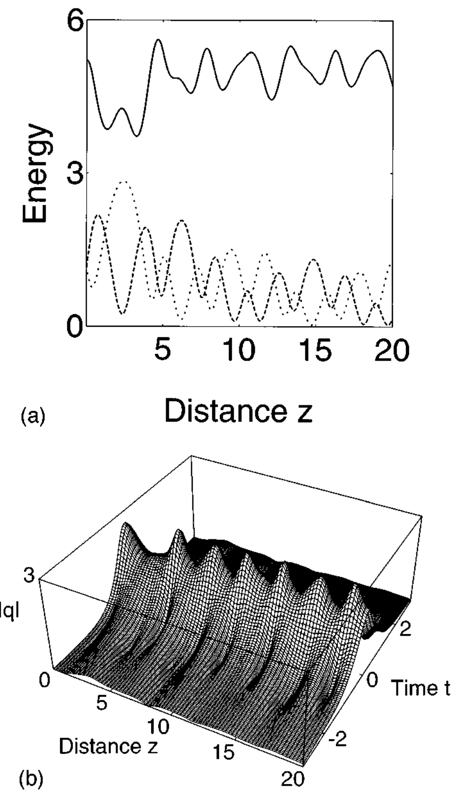

FIG. 7. The same as in Fig. 2(a), but with the following initial conditions: $E=\sqrt{45}$, $K=0.5$, $q^{(n)}=A_n\mathrm{sech}(A_n t)$ $\exp(ip_n)$, $A_1=A_3=0.4E/4$, $A_2=0.6E/2+0.6$, $p_1=-p_3=\pi/2$, $p_2=0$. The dashed (dotted) line represents the energy in the guide 3 (1); the energy remains confined in the central waveguide (solid line). (b) Corresponding propagation of the localized pulse in the central waveguide $q=q^{(2)}$.

$\sqrt{8\pi K}-\sqrt{24K}\sim 0.114\sqrt{K}$, and thus it could in principle be increased to compensate radiation losses by increasing K. However, an increase of K would induce a loss of the localized stable solution, because the coupling is enhanced, unless the input energy grows to keep the system in the same conditions. Note that the governing equation can always be scaled as $q_K^{(n)}=\sqrt{K}q^{(n)}$, $\tau=\sqrt{K}t$, and $\xi=Kz$ and thus the parameter which fixes the dynamics is the ratio K/E^2 [31]. Thus by increasing K linearly we increase the margin of a $\sqrt{K}$ factor and must increase the initial energy of a squared value to keep stability. This is not practical and also radiated energy can be expected to increase too at the same rate.

We thus conclude that the smallness of the energy gap among stable solutions hampers the steering. Another possibility would be to increase only the initial stored energy, but this would render a much more stable localized state. Nonetheless, if a broader distribution among the waveguides of the increased energy is allowed, steering can be reached by means of a phase symmetry breaking initial condition, as we explain below.

Let us consider again the symmetric case, represented in the phase portrait of Fig. 1. As the total energy increases, we find that the region of localized solutions expands and thus there exist stable states for which a is less than 1 (though still close); this means that a part of the energy is shared with the other waveguides. Initial states of this kind seem good

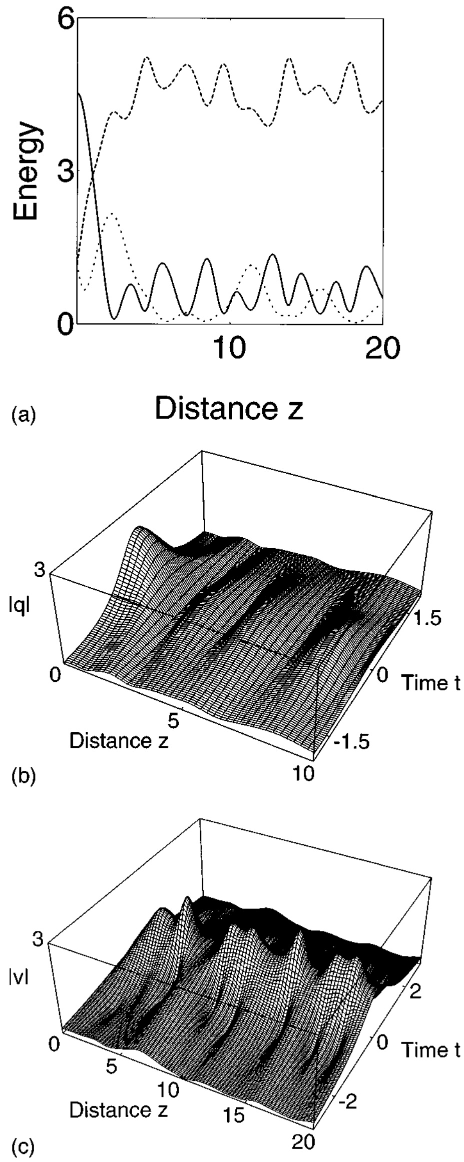

FIG. 8. (a) The same as in the preceding figure except that $A_2 = 0.6E/2 + 0.25$. The pulse from the waveguide $2(q = q^{(2)})$ steers to 3 ($v = q^{(3)}$), as is clearly observable in (b) and (c). Note in (a) that only a small amount of energy crosses to the waveguide 1.

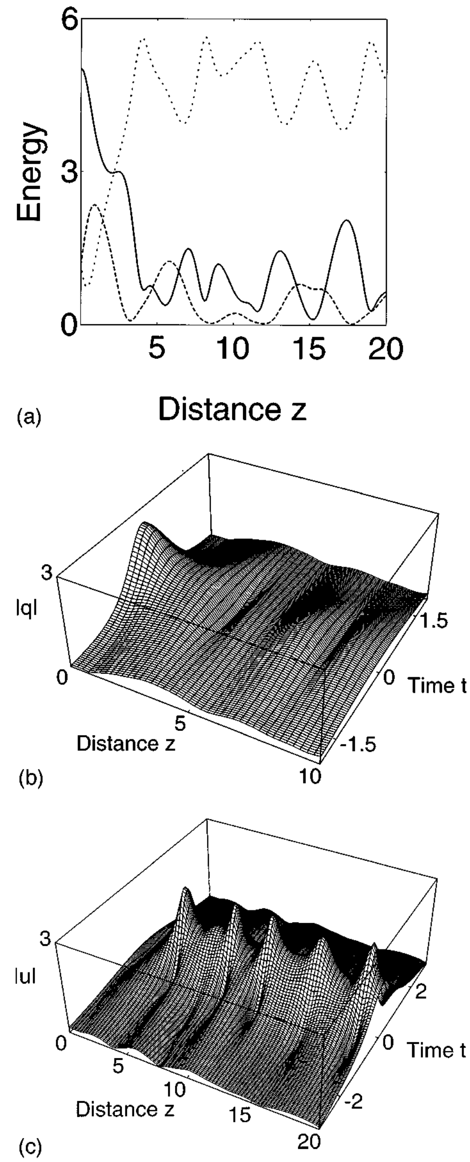

FIG. 9. The same as in the previous two figures, except that $A_2 = 0.6E/2 + 0.5$. There exists a small set of initial conditions for A_2 close to this value for which the steering takes place toward the guide 1 ($u = q^{(1)}$), as can be seen in (b) and (c).

candidates to realize switching, since more energy can be stored, to compensate radiation losses, but the system is still close to the separatrix.

We start with a localized solution where most but not all of the energy is concentrated in the central waveguide. The total energy is chosen to be $\sqrt{45}$, i.e., larger than the threshold of stability ($\sqrt{8\pi K}, K = 1$). By means of Eqs. (9) we can calculate exactly the position of the saddle point of Fig. 1 which is $a_s \sim 0.6995$, $p_s = \pi$. Upon substitution of the value of the saddle into Eq. (10) we can calculate $h(a_s, p_s)$ which will be conserved along the homoclinic orbit. In particular, we can find the value of a on the separatrix at $p = 0$; this is the value at which a is closer to unity, i.e., the most localized state. In our example we found $a_{\max} \sim 0.96$; hence, if more than 96% of the energy is concentrated into the central guide, localization persists. Below threshold and according to the model, the onset of the coupling dynamics should be found, leading to the splitting of the energy on the sides. This is confirmed by the result of the numerical integrations presented in Figs. 5 and 6. This case closely resembles the behavior of the NLDC, where a similar transition to the coupling dynamical regime has been widely reported [1,4,22].

In this symmetric case, no final steady state is reached since, as said, we need a symmetry breaking perturbation to get tristable operational mode. Guided by previous results [11,18–20,32] we try to apply a linear phase shift among adjacent waveguides in order to achieve the energy transfer in a direction prefixed by the slope of the shift. The energy now is expected to move mostly to one side and there get trapped when it is above the required threshold. This is shown in Fig. 7, where an asymmetric stable state is presented, and Fig. 8, where the energy steering to a new stable condition is found by simply decreasing the input energy in the central waveguide E_q. Note that in this case we had to

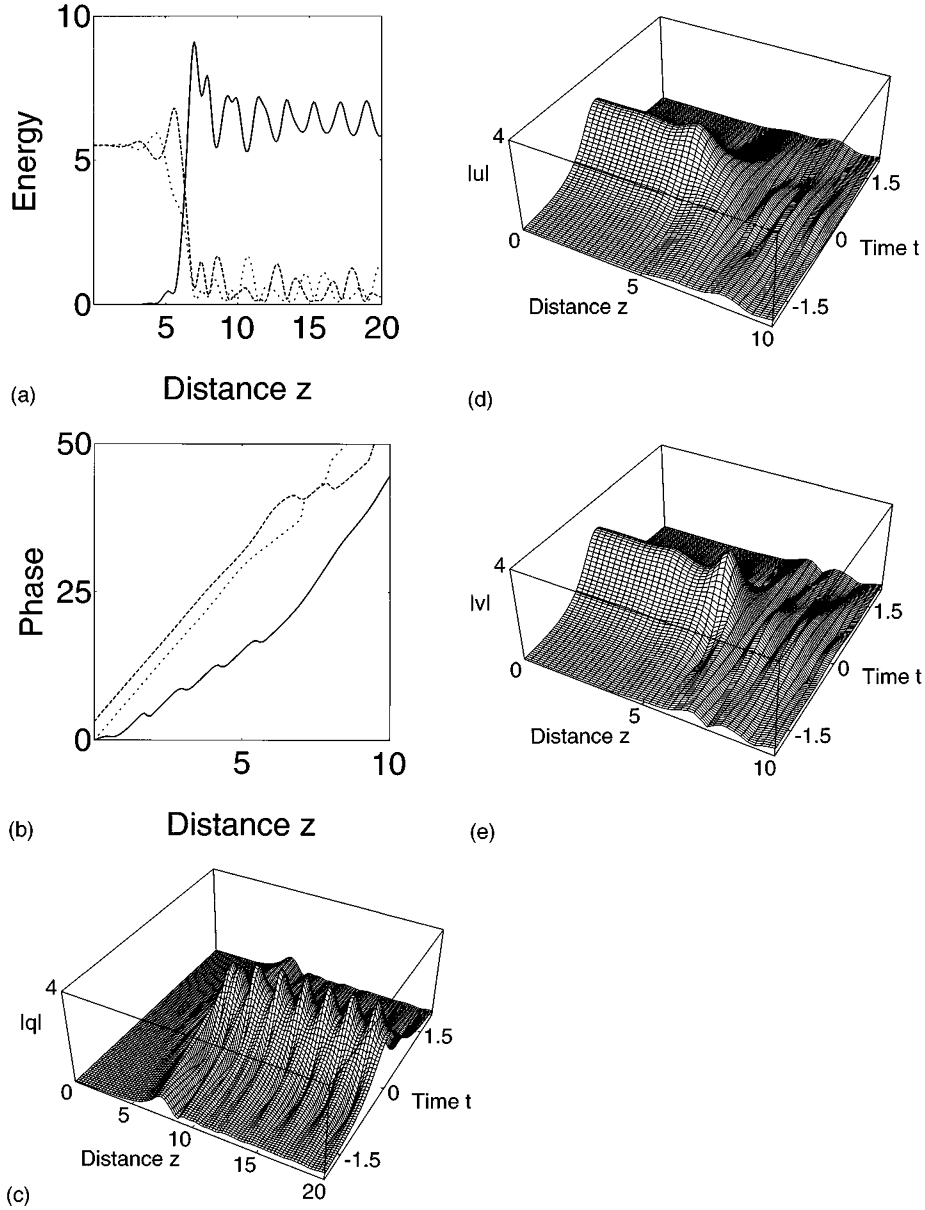

FIG. 10. (a) Evolution of the energy for a slight perturbation of an antisymmetric solution: $E=\sqrt{8\pi K}+4$, $K=1$, $q^{(n)}=A_n\mathrm{sech}(A_n t)\exp(ip_n)$, $A_1=A_3=0.999E/4$, $A_2=0.001E/2$, $p_1=p_2=0$, $p_3=\pi$, of Eqs. (1) (dotted, $u=q^{(1)}$; solid; $q=q^{(2)}$; dashed, $v=q^{(3)}$). Note (b) that initially the side pulses (u,v) are phase locked to π; this antisymmetry is broken when the large energy transfer occurs among the waveguides. Input and output pulses in all the waveguides present a smooth behavior and the emission of radiation mainly takes place at the point of transfer (c), (d), (e).

allow an even broader distribution with respect to what is predicted by our previous analysis. This is expected if we think that the model (9) is based on the symmetric input hypothesis, which is not the case of Figs. 7 and 8. By using the general set of equations (6) we found that a broader energy distribution is needed to break stability and this indication has been used to get the result of Figs. 7 and 8. Note also that K had to be lowered; for larger K we observed quicker but not stable energy transfers. In the small layer of energies among the two values shown in Figs. 7 and 8 we happened to find the numerical evidence of the existence of a small, though clearly defined, set of initial conditions which leads to the steering of the stable state towards the opposite waveguide. An example is shown in Fig. 9. This fact is very promising in view of an application of these devices in all-optical switching. Note in fact that the tristable switching is solely controlled by changing the energy in the central waveguide. If this is large, localization is the only outcome; for intermediate energy levels we found steering on both sides, and for low energies coupling prevails.

Even more remarkable is the fact that this interpretation of steering and trapping by means of stability energy levels may be extended to larger arrays, thus explaining why the steering of localized states, besides a symmetry breaking initial condition, needs also large energies with broad distributions, as found numerically in previous works [11,18–20]. In fact, to steer a localized state to another position of an array requires an initial energy well above the threshold of stability of the final, desired state, since a lot of it is lost in radiation during the steering process. Moreover, this additional energy

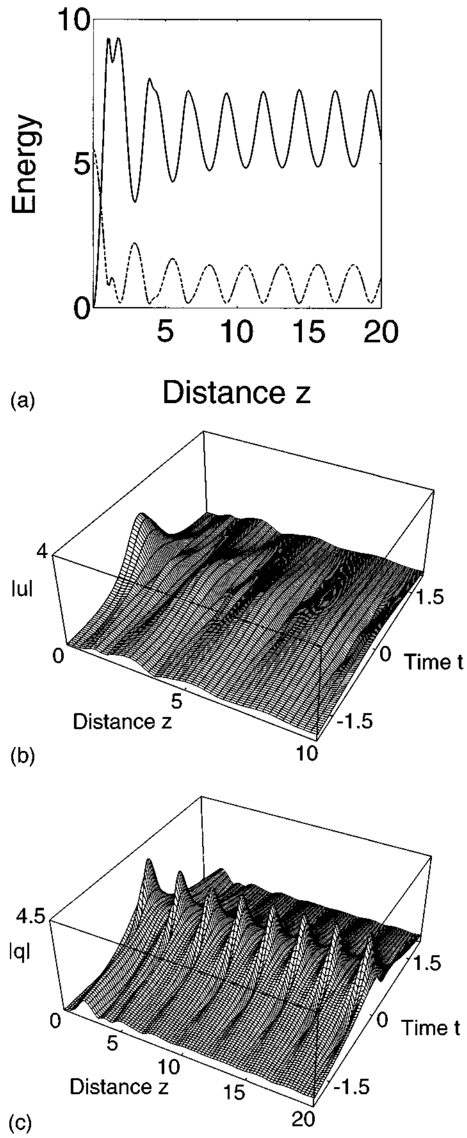

FIG. 11. (a) The same as in the preceding figure but with $p_3=0$. The final localized state is reached much quicker than in the case of the antisymmetric initial condition. The input energy is less than that of Fig. 2 and 3 and thus less energy is emitted when the pulse forms and a smoother behavior is observed in the pulse propagation (b), (c).

cannot be initially stored in the same waveguide, since it would strengthen the stability of the initial state. In order to keep the system close to the edge of stability, thus allowing a weak perturbation to realize the switching, a broader distribution must be used.

As a matter of fact, what ultimately prevents the potential multistable operation of waveguide arrays is the fact that states for which the energy is localized in a single waveguide, in an array of identical waveguides, have approximately the same energy threshold of stability. Unless this symmetry is broken, steering would require large distributions of energy among waveguides, as experiment seems to demonstrate [13]. Hence a really multistable switching among well localized solutions seems to be excluded *a priori*. However, we want to conclude this section with a possible suggestion to overcome this problem. As we noted in the preceding section, the creation of a tunable energy gap can be obtained in the nonlinearly asymmetrical coupler; a natural extension would be to realize a structure in which a stair of energy levels is realized, i.e., $E_{cr0}>E_{cr1}>E_{cr2}>\cdots$, by means of increasing step by step the nonlinearity of the cores. Such a device might be able to attain multistable, cw or pulsed, switching by simply controlling the input energy. Thus the design and realization of novel devices, such as planar waveguides made of layers of semiconductor with different stoichiometry or coupled glass fiber arrays doped with different volume fractions of semiconductors, might disclose interesting experimental and technical applications in the all-optical signal processing.

ANALYSIS OF THE ANTISYMMETRIC EXACT SOLUTION

The study of unstable, exact solutions of the NLDC [22] has revealed interesting dynamics and possibly more reliable applications of that device. For this reason we finally address the behavior of an exact solution of the PDE's (1), i.e.,

$$q^{(1)}(z,t)=-q^{(3)}(z,t)=A\mathrm{sech}(At)\exp(ip),$$

$$q^{(2)}(z,t)=0. \tag{18}$$

This analytical solution is antisymmetric and resembles the localized state, previously discussed, where the energy is concentrated in the sides. In fact, if we consider the solution of Eqs. (6) which corresponds to Eq. (18), we have $A_1=A_3=A=E/4$, $A_2=0$, $p_+(z)=-A^2z/2$, and $p_-(z)=p_+(z)-\pi$ which, apart form the phase difference π (instead of 0) among p_+ and p_-, looks like the symmetric one. However, its properties are quite different as the simplified model of Eqs. (6) indicates itself. By studying more carefully the system (6) under this initial condition, we can reveal an important difference in the equations for the phases. In fact the term proportional to K is zero in the antisymmetric case, but not for the symmetric solution. In the latter case, as the input energy decreases, the first term and that depending on K become comparable, as previously noted, and the localized state becomes unstable, while for the antisymmetric solution, the term proportional to K remains zero and thus p_+ and p_- stay out of phase during propagation since $dp_+/dz=dp_-/dz$ for all z. The energy content in the side waveguides changes with propagation since the π difference phase implies that $dA_1/dz\neq dA_3/dz$ but the transfer to the central waveguide is very slow because the phase difference is locked to π and this causes dA_2/dz to remain very small.

In effect, captured by the variational approach, is a feature of the governing equation too, if an antisymmetric perturbation is applied to this kind of solution. This can be noticed in Fig. 10, where we show the results of numerical integration of Eq. (1) with an initial condition of the type (18). Note that the pulses are initially phase locked to π, as predicted, and that at the output we get, once more, a localized stable solution, since the energy of the new pulse is larger than the threshold level. However, the dynamics that leads to the transfer is now much slower than in the case of the symmetric solution, as can be observed by comparing Fig. 10 with Fig. 11. Hence with the same initial energy of Fig. 10 we might obtain, with symmetric initial conditions, a quicker transfer and localization of energy in the central waveguide. This effect might be exploited for switching if a short, say 3 or 4 length units, device is used. Besides the shorter length, the advantage of this device, with respect to the long device of Figs. 2 and 3, is that less initial energy is needed. This decrease in the input energy also contributes to a quicker and smoother formation of the output pulse, as noted by comparing Figs. 10 and 11 with Figs. 2 and 3.

CONCLUSIONS

We studied the bistable and tristable soliton switching in collinear arrays of three, linearly coupled, nonlinear waveguides. By means of a variational method we found that different energy thresholds of stability are associated to two localized solutions of the device. The gap among these energy levels is large enough to compensate radiation loss and thus allows the bistable operation of the device. Numerical solutions of the governing equations confirm the analytical predictions.

These results can be extended to an asymmetrical nonlinear directional coupler, with unequal nonlinear coefficient. The analysis predicts that, through the enhancement of the nonlinear coefficient in one of the waveguides, the energy gap would be minimal and bistable operation would be attained at the lowest energy level. The physical energy value is also expected to be low because of the concomitant increase of the nonlinearity and of the device length. Semiconductor planar waveguides and semiconductor doped glass waveguides have been identified as a possible candidate for an experimental realization.

Guided by the reduced set of ODE's, obtained through the variational approach, we then explored the tristable mode of operation, demonstrating the stable-to-stable switching of pulses among adjacent waveguides is highly hampered by the small separation of the energy thresholds for the stability among the initial and the final state. However, the model indicates that this switching is still possible if the initial energy is divided among the waveguides. The broader distribution, in fact, allows the system to store more energy though staying still close to the edge of stability. Thus a symmetry breaking perturbation can induce the steering of the pulse towards one side. This prediction has been confirmed numerically and we found that steering in both directions can be accomplished by simply varying the input energy in the central waveguide.

This result has been applied to explain why the steering of localized states has proved to be a difficult task in large coupled arrays. In fact, the steering is submitted to the following requirements: the total initial energy must be well above the threshold of stability of the final, desired state, as a compensation of radiation losses during the steering; a broader distribution is needed to store this additional energy while keeping the system close to the homoclinic orbit which separates different regimes; finally a symmetry breaking perturbation, such as a differential phase, must be applied to ''kick'' the energy in a prefixed direction of the array. Novel multistable devices, based on designed asymmetries in the nonlinear coefficient of the cores, have also been suggested on the basis of the analysis.

Eventually we have studied a particular solution of the governing equation, which presents a peculiar slow dynamics, which is captured by the variational model. Application for switching in short length arrays is predicted.

The physical and technical results presented in this paper are thought to be an improvement of the knowledge of the dynamics of arrays of coupled waveguides. Henceforth, they are highly relevant in the design of reliable devices for all-optical signal processing.

ACKNOWLEDGMENTS

Both authors would like to express their gratitude to the Division of Applied Mathematics, Brown University for their hospitality while this work was being conducted. This research was sponsored by the Air Force Office of Scientific Research, Air Force Materials Command, USAF, under Grant Number F49620-94-1-0007. The work of M.S. was also supported by a grant from the University of Padova (Italy).

[1] S. Trillo, S. Wabnitz, E.M. Wright, and G.I. Stegeman, Opt. Lett. **13**, 672 (1988).
[2] G.I. Stegeman, E.M. Wright, N. Finlayson, R. Zanoni, and C.T. Seaton, J. Lightwave Technol. **6**, 953 (1988); A. Villeneuve, C.C. Yang, P.G.J. Wigley, G.I. Stegeman, J.S. Aitchinson, and C.N. Ironside, Appl. Phys. Lett. **61**, 147 (1992).
[3] F.Kh. Abdullaev, R.M. Abrarov, and S.A. Darmanyan, Opt. Lett. **14**, 131 (1989).
[4] C. Paré and M. Florjańczyk, Phys. Rev. A **41**, 6287 (1990).
[5] E. Cagliotti, S. Trillo, S. Wabnitz, B. Crosignani, and P. Di Porto, J. Opt. Soc. Am. B **7**, 374 (1990).
[6] P.L. Chu, B.A. Malomed, and G.D. Peng, J. Opt. Soc. Am. B **10**, 1379 (1993).
[7] Y.S. Kivshar, Opt. Lett. **18**, 7 (1993).
[8] I.M. Uzunov, R. Muschall, M. Gölles, Y.S. Kivshar, B.A. Malomed, and F. Lederer, Phys. Rev. A **51**, 2527 (1995).
[9] H.A. Haus and L. Molter-Orr, IEEE J. Quantum Electron. **QE-19**, 840 (1983).
[10] C. Schmidt-Hattenberger, U. Trutschel, and F. Lederer, Opt. Lett. **16**, 294 (1991).
[11] A.B. Aceves, C. De Angelis, T. Peschel, R. Muschall, F. Lederer, S. Trillo, and S. Wabnitz, Phys. Rev. E **53**, 1172 (1996).
[12] D.B. Mortimore and J.W. Arkwright, Appl. Opt. **30**, 650 (1991).
[13] P. Millar, J.S. Aitchinson, J.U. Kang, G.I. Stegeman, and A. Villeneuve (unpublished).
[14] N. Finlayson and G.I. Stegeman, Appl. Phys. Lett. **56**, 2276 (1990).
[15] J.M. Soto-Crespo and E.M. Wright, J. Appl. Phys. **70**, 7240 (1991).
[16] N.N. Akhmediev and A.V. Buryak, J. Opt. Soc. Am. B **11**, 804 (1994).

[17] A.V. Buryak and N.N. Akhmediev, IEEE J. Quantum Electron. **QE-31**, 682 (1995).
[18] A.B. Aceves, C. De Angelis, S. Trillo, and S. Wabnitz, Opt. Lett. **19**, 332 (1994).
[19] A.B. Aceves, C. De Angelis, A.M. Rubenchik, and S.K. Turitsyn, Opt. Lett. **19**, 329 (1994).
[20] A.B. Aceves, C. De Angelis, G.G. Luther, A.M. Rubenchik, and S.K. Turitsyn, Physica D **87**, 262 (1995;).
[21] A.B. Aceves, M. Santagiustina, and C. De Angelis, J. Opt. Soc. Am. B (to be published).
[22] N.N. Akhmediev and J.M. Soto-Crespo, Phys. Rev. E **49**, 4519 (1993).
[23] A.V. Buryak and N.N. Akhmediev, Opt. Commun. **110**, 287 (1994).
[24] A.B. Aceves and M. Santagiustina (unpublished).
[25] N. Akhmediev and A. Ankiewicz, J. Elec. Electr. Eng., Australia **15**, 277 (1995); Phys. Rev. Lett. **70**, 2395 (1993).
[26] B.A. Malomed, I.M. Skinner, P.L. Chu, and G.D. Peng, Phys. Rev. E **53**, 4084 (1996).
[27] G.R. Hadley, Opt. Lett. **16**, 624 (1991); IEEE J. Quantum Electron. **28**, 363 (1992).
[28] R.K. Jain and R.C. Lind, J. Opt. Soc. Am. **73**, 647 (1983).
[29] M. Sheik-Bahae, D.C. Hutchings, D.J. Hagan, and E.W. Van Stryland, IEEE J. Quantum Electron. **27**, 1296 (1991).
[30] C. Flytzanis, F. Hache, M.C. Klein, D. Richard, and P. Roussignol, Prog. Opt. **29**, 323 (1991).
[31] M.I. Weinstein and B. Yeary, Phys. Lett. A **222**, 157 (1996).
[32] S. Trillo and S. Wabnitz, Opt. Lett. **16**, 1 (1991).